MATLAB Notes for MA202

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These are class notes for MA202 (Mathematical Analysis IV). For more advanced methods and basics of programming in MATLAB see the notes for Differential Equations (MA320), Mathematical Modeling (MA422) and Special Topics in Mathematics (MA360) on my website: http://www.usciences.edu/~lvas

Content

0. Review of Matlab notes for MA201

0.1 Representing Functions. Solving Equations.

MATLAB can perform basic arithmetic operations. You can use +, -, *, \ and ^ to add, subtract, multiply, divide or exponentiate, respectively.

To perform symbolic calculations in MATLAB, use **syms** to declare the variables you plan to use. For example, suppose that you need factor x^2-3x+2. First you need
>> syms x (you are declaring that *x* is a variable)

Then you can use the command **factor**.
>> factor(x^2-3*x+2)
ans =
(x-1)*(x-2)

Note that we entered **3*x** to represent 3*x* in the command above. **Entering * for**

multiplication is always necessary in MATLAB.

For solving equations, you can use the command **solve**. This command has the following form:

solve('*equation***', '***variable for which you are solving***')**

For example, let us solve the equation $3x^2-8x+2=0$.

>> solve('3*x^2-8*x+2=0','x')
ans =
[4/3+1/3*10^(1/2)]
[4/3-1/3*10^(1/2)]

If we want to get the answer in the decimal form with, say, three significant digits, we can use the command **vpa**.

>> vpa(ans, 3)
ans =
[2.38]
[0.28]

By changing 3 in the command **vpa(ans, 3)** you can specify the number of digits in the answer.

You can solve an equation in two variables for one of them. For example the command

>> solve('y^2-5*x*y-y+6*x^2+x=2', 'y')

solves the given equation for values of *y* in terms of *x*. The answer is:

ans =
[3*x+2]
[2*x-1]

You can solve more than one equation simultaneously. For example suppose that we need to solve the system $x^2+ x+ y^2 = 2$ and $2x-y = 2$. We can use:

>> [x,y] =solve('x^2+ x+ y^2 = 2', '2*x-y = 2')

to get the answer

x =
[2/5]
[1]
y =
[-6/5]
[0]

Note that the **[x,y]=** part at the beginning of the command was necessary since without it MATLAB produces the answer:

ans =
x: [2x1 sym]
y: [2x1 sym]

This answer tells us just that the solutions are two values of the pair (x,y) but we do not get the solutions themselves. To get the solution vectors displayed, we must use **[x,y]=** before the command **solve**.

The following table gives an overview of how most commonly used functions or expressions are represented in MATLAB.

function or symbol	representation in MATLAB
e^x	exp(x)
ln x	log(x)
log x	log(x)/log(10)
log. base a of x	log(x)/log(a)
sin x	sin(x)
cos x	cos(x)
arctan(x)	atan(x)
π	pi

To represent a function, use the command **inline**. This command has the following form:

inline('*function***', '***independent variable of the function***')**

Here is how to define the function x^2+3x-2:

```
>> f = inline('x^2+3*x-2', 'x')
f =
   Inline function:
   f(x) = x^2+3*x-2
```

After defining a function, we can evaluate it at a point. For example,

```
>> f(2)
ans =     8
```

Just as when using calculator, one must be careful when representing a function. For example

$\frac{1}{x(x+6)}$ should be represented as **1/(x*(x+6))** not as **1/x*(x+6)** nor as **1/x(x+6),**

$\frac{3}{x^2+5x+6}$ should be represented as **3/(x^2+5*x+6)** not as **3/x^2+5*x+6,**

e^{5x^2} should be represented as **exp(5*x^2)** not as **e^(5*x^2), exp*(5*x^2), exp(5x^2)** nor as **exp^(5*x^2).**

0.2 Basic Graphing

When graphing functions depending on a variable *x,* you can start your session by declaring x for your variable

```
>> syms x
```

The simplest way to graph a function is to use the command **ezplot** (easy plot). For example, to graph the function x^2+x+1, we use:

```
>> ezplot(x^2+x+1)
```

If we want the variable x to take values between -2 and 2, we use:

>> ezplot(x^2+x+1, [-2, 2])

If we want to see the graph for *x* in the interval [-2, 2] and for *y* in the interval [1, 4], we type
>> axis([-2 2 1 4])

To plot multiple curves on the same window, you can also use the **ezplot** command. For example:
>> ezplot('sin(x)')
>> hold on
>> ezplot('exp(-x^2)')
>> hold off

0.3 Differentiation

To differentiate a function, we use the command **diff**. For example,
>> diff(x^3-2*x+5)
ans = 3*x^2-2

Note that you need to declare x as your variable using **syms x** if you have not done so before.

To get the second derivative, use:
diff(x^3-2*x+5, 2)
ans = 6*x

0.4 Integration

We can use MATLAB for computing both definite and indefinite integrals using the command **int**. For the indefinite integrals, consider the following example:
>> int(x^2)
ans = 1/3*x^3

To obtain the definite integral of the same function in the bounds 0 to 1 use:
>> int(x^2, 0, 1)
ans = 1/3

1. Vectors

You can enter a vector by typing a list of numbers separated by either commas or spaces inside the square brackets. For example,
>> X = [1,2,3]
>> Y = [4 5 6]

Then you can perform the usual operations on vectors:

>> X+Y (addition)
ans = 5 7 9

>> 3*X (scalar multiplication)
ans = 3 6 9

>> dot(X,Y) (dot product)
ans = 32

>> cross(X,Y) (cross product)
ans = -3 6 -3

>> norm(X) (norm or length of a vector)
ans = 3.7417

2. Differentiation of Multi-variable Functions

The command **diff** can be used to compute **partial derivatives**. For example, to find the first partial derivative with respect to y of the function x^2y^3, we use

>> syms x y
>> diff(x^2*y^3, y)
ans = 3*x^2*y^2

To find the second partial derivative with respect to *x*, we use

>> diff(x^2*y^3, x, 2)
ans = 2*y^3

For the mixed second partial derivative, we can use:

>> diff(diff(x^2*y^3, x), y)
ans = 6*x*y^2

To get the derivative of z= $3x^9y^7$ twice with respect to x and three times with respect to y, you can use

>> diff(diff(3*x^9*y^7, x, 2), y, 3)

Practice problems 1

1. Determine if each pair of vectors is orthogonal, parallel or neither.
   X = [1, -3, 4], Y = [2, 5, -3]
   X = [1, -3, 4], Y = [-3, 9, -12]
   X = [1, -3, 4], Y = [-2, 2, 2]

Find the length of vector **X**.

2. Find the first partial derivatives of the function $\frac{\sin xy}{\ln(x^2+1)} - e^y$.
3. Find all of the first and second partial derivatives of the function $e^{x^2}\sin xy$.

3. Curves and Surfaces in 3D

3.1 Parametric Plots

We can use the command **ezplot** to graph a parametric curve as well. For example, to graph a circle *x* = cos *t, y* = sin *t* for 0 ≤ *t* ≤ 2*π,* we have:

>> ezplot('cos(t)', 'sin(t)', [0, 2*pi])

3.2 Curves in Three-Dimensional Space

The command for drawing 3D curves is **ezplot3**. For example, suppose that we need to graph the helix *x* = cos *t, y* = sin *t, z* = *t,* for -10 ≤ *t* ≤ 10.

>> ezplot3('cos(t)', 'sin(t)', 't', [-10, 10])

3.3 Surfaces in Three-Dimensional Space

There are two commands for graphing surfaces: **ezmesh** and **ezsurf**. The first produces a transparent wired plot, a mesh surface. The second produces a shaded, nontransparent surface. Both commands can be used for graphing surfaces that are given in the form

z = f(x, y)

For example, the graph of the cone z = $\sqrt{x^2+y^2}$ over the square -7 ≤ *x* ≤ 7, -7 ≤ *y* ≤ 7, can be obtained as follows:

>> ezmesh('sqrt(x^2+y^2)', [-7, 7], [-7, 7])
Using the **ezsurf** command: **>> ezsurf('sqrt(x^2+y^2)', [-7, 7], [-7, 7])**

3.4 Parametric Surfaces in Three-Dimensional Space

Ezmesh and ezsurf are also used when graphing the surfaces that are given in parametric form

x = f(s, t)
y = g(s, t)
z = h(s, t)

The cone z = $\sqrt{x^2+y^2}$ can be represented by parametric equations as

$$x = r\cos t$$
$$y = r\sin t$$
$$z = r$$

Using this representation, we can get the graph as follows
>> ezmesh('r*cos(t)', 'r*sin(t)', 'r', [0 , 10, 0, 2*pi])

or using the **ezsurf** command: **>> ezsurf('r*cos(t)', 'r*sin(t)', 'r', [0 , 10, 0, 2*pi])**

3.5 Vector Fields

We can plot the vector field **F**(*x, y*) = *P(x, y)* **i** + Q(x, *y)* **j** by using the command **quiver(x, y, P(x,y), Q(x,y))**.

For example, we plot the vector field **F**(*x, y*) = *x* **i** - *y* **j** as follows:
>> [x,y]=meshgrid(-1:.2:1, -1:.2:1);
>> quiver(x, y, x, -y);
>> axis equal

In 3D, we can also plot the vector field

F(*x, y, z*) = *P(x, y, z)* **i** + Q(x, *y, z)* **j** + *R(x, y, z)* **k**

by using the command **quiver3(x, y, z, P(x,y,z), Q(x,y,z), R(x,y,z)).**

For example, we plot the field **F**(*x, y, z*) = *x* **i** + *y* **j** + *z* **k** as follows:
>> [x,y,z]=meshgrid(-3:1:3, -3:1:3, -3:1:3);
>> quiver3(x, y, z, x, y, z);
>> axis equal

We can plot the gradient field of the surface *F*(*x, y, z*)=0 by using the command **surfnorm.** For example to get the gradient field of the surface *z=xe^(-x^2-y^2)* together with the graph of surface itself, you can use:

```
[X,Y] = meshgrid(-2:0.25:2,-1:0.2:1);
Z = X.* exp(-X.^2 - Y.^2);
[U,V,W] = surfnorm(X,Y,Z);
quiver3(X,Y,Z,U,V,W,0.5);
hold on
surf(X,Y,Z);
colormap hsv
view(-35,45)
```

axis ([-2 2 -1 1 -.6 .6])
hold off

<u>Practice problems 2</u>

1. Graph the parametric curve $x = t \cos t$, $y = t \sin t$ for $0 \le t \le 10\pi$.
2. Graph the cylinder $x^2 + y^2 = 1$.
3. Write down the parametric representation of the curve of the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $y + z = 2$. Graph this curve using this representation. Then graph the cylinder and the plane on the same graph.
4. Graph the paraboloid $z = x^2 + y^2$ over the square $-6 \le x \le 6$, $-6 \le y \le 6$.
5. Give the parametric representation of the above paraboloid using the cylindrical coordinates. Graph it using this representation.
6. Graph the sphere of radius 3 centered at origin.
7. Graph the vector field $\mathbf{F}(x, y, z) = y\,\mathbf{i} + x\,\mathbf{j} + z^2\,\mathbf{k}$

3.6 Special effects

You can change the title above the graph by using the command **title**. For example,

>> ezplot('x^2+2*x+1')
>> title 'a parabola'

You can add labels to *x* and *y* axis by typing **xlabel** and **ylabel.**

You can add text to your picture. For example, suppose that we want to add a little arrow pointing to the minimum of this function, the point (-1, 0). We can do that with the command:

>> text(-1, 0, '(-1, 0) \leftarrow a minimum')

We can also make three-dimensional plots nicer by
- using perspective (**camproj('perspective')**; undo by **camproj('orthographic')**)
- showing bounding box (**box on**; undo by **box off**),
- making axis units equal (**axis tight, axis equal**; undo by **axis normal**),
- showing grid (**grid on**; undo by **grid off**) and
- allowing rotation by mouse dragging (**rotate3d on**; undo by **rotate3d off**).
For example,

>> camproj('perspective');
>> box on;
>> axis tight;
>> axis equal;
>> grid on;
>> rotate3d on;
>> ezsurf('sqrt(x^2+y^2)', [-7, 7], [-7, 7])

You can produce an animated picture with **comet**. This command produces a parametric plot of a curve just as **ezplot** does, except that you can see the curve being traced out in time. For example, we can trace the motion on the circle *x* = cos *t, y* = sin *t* by using

>> t = 0:0.1:4*pi; (meaning that t has values between 0 and 4π, 0.1 step away from each other)
>> comet(cos(t), sin(t))

If the point is moving too fast, you can reparameterize the same circle as follows
>> t = 0:0.1:200*pi;
>> comet(cos(t/50), sin(t/50))

<u>Practice problems 3</u>

1. Graph the cubic curve $y = x^3$. Label the axis and put a title "a cubic curve" on the top of the graph. Indicate on the graph that the point (0,0) is an inflection point.
2. Using all of the above commands for nicer three-dimensional graphics, graph the paraboloid $z=x^2+y^2$.
3. Trace the curve $x = t\cos(t/20)$, $y = t\sin(t/20)$ in time for $0 \le t \le 200\pi$.

4. INTEGRATION OF MULTI-VARIABLE FUNCTIONS

Suppose that we want to evaluate the integral $\int_{-1}^{1}\int_{2x^2}^{1+x^2}(x+2y)\,dy\,dx$.

In MATLAB we can do that as follows.

>> syms x y
>> int(int(x+2*y, y, 2*x^2, 1+x^2), x, -1, 1)

ans = 32/15

Similarly, we can evaluate the triple integrals. For example, to evaluate $\int_0^1\int_0^x\int_0^{xy} 6xyz\,dx\,dy\,dz$ use
>> int(int(int(6*x*y*z, z, 0, x*y) y, 0, x), x, 0, 1)
The integral $\int_0^1\int_y^{2y}\int_y^{y+z} 2y\sin x\,dx\,dy\,dz$ could be evaluated as
>> int(int(int(2*y*sin(x), x, y, y+z) z, y, 2*y), y, 0, 1)

<u>Practice problems 4</u>

1. Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $x = y^2$.

2. Evaluate the integral $\int_0^1 \int_0^z \int_0^{x+z} 6xzdydzdx$.

5. SERIES. TAYLOR SERIES

In MATLAB, we can evaluate finite and infinite sums. Suppose that we want to find the numerical value of $\sum_{n=1}^{10} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

For the first sum, we have:

>> syms n
>> symsum(1/n^2, 1, 10)
ans = 1968329/1270080

For the second sum:

>> symsum(1/n^2, 1, inf)
ans = 1/6*pi^2

MATLAB can manipulate symbolic series as well. For example, for $\sum_{n=0}^{\infty} a^n$ where -1 < *a* < 1, we get

>> syms n a
>> symsum(a^n, 0, inf)
ans = -1/(a-1)

We can use the command **taylor** to generate Taylor polynomial of specified order at a specified point.

For example, for the Taylor polynomial of e^x of order 4 at 0, we have

>> syms x
>> taylor(exp(x), x, 4)
ans = 1+x+1/2*x^2+1/6*x^3

For the Taylor polynomial of the same function of order 4 at 3, we have

>> taylor(exp(x), 4, 3)
ans = exp(3)+exp(3)*(x-3)+1/2*exp(3)*(x-3)^2+1/6*exp(3)*(x-3)^3

<u>Practice problems 5</u>

1. Using MATLAB determine whether the following series is convergent or divergent.

   $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ $\qquad \sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

2. Find the Taylor polynomial of order 5 at 0 and at 2 for the function sin *x*.
3. Find the Taylor polynomial of order 5 at infinity for the function $e^{1/x}$.