Math 102
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Optimization Problems

To solve an applied optimization problem:

1. Sketch a diagram if possible.
2. Select the variables to represent the independent variables and the quantity to be maximized or minimized.
3. Write down the equation for the quantity to be minimized or maximized, **objective equation**. Write down the equation that relates the independent variables, **constraint equation**. Using the constraint, eliminate second variable from the objective equation.
4. Find the minimum or maximum.
5. Interpret the solution. Write a sentence that answers the question posed in the problem.

Practice Problems.

1. Find the dimensions of the rectangular garden of greatest area that can be fenced off with 400 feet of fencing.

2. Consider a box with a square base. Find the dimensions of the box with the surface area 96 square inches, such that the volume is as large as possible.

3. The Hardy - Weinberg Law states that the proportion of individuals in a population who are heterozygous is $2pq$ and the proportion of individuals who are homozygous is $p^2 + q^2$. Recall that $p + q = 1$.
   a) Find the maximal percentage of people that are heterozygous.
   b) Find the minimal percentage of people that are heterozygous.
   c) Find the minimal percentage of people that are homozygous.
   d) Find the maximal percentage of people that are homozygous.

4. The concentration of a certain medication in a patient's bloodstream can be given by
$$C(t) = \frac{5.3t}{t^2 + 4t + 5} \quad 0 \leq t \leq 8$$
where $C(t)$ is in milligrams per cubic centimeter and t is the number of hours after the medication has been administered. How many hours after the medication has been administered is the concentration at a maximum? What is the maximum concentration?

5. An open top box is made with a square base and should have a volume of 6000 cubic inches. If the material for the sides costs \$.20 per square inch and the material for the base costs \$.30 per square inch, determine the dimensions of the box that minimize the cost of the materials.

6. A fence must be built in a large field to enclose a rectangular area of 400 square meters. One side of the area is bounded by existing fence; no fence is needed there. Material for the fence cost \$ 8 per meter for the two ends, and \$ 4 per meter for the side opposite the existing fence. Find the cost for the least expensive fence.

7. A company wishes to manufacture a box with a volume of 36 square feet that is open on the top and is twice as long as it is wide. Find the dimensions of the box produced from the minimal amount of the material.

8. In a physics experiment, temperature T (in Fahrenheit) and pressure P (in kilo Pascals) have a constant product of 5000. You are monitoring a function $F = T^2 + 50P$. Find the temperature T and pressure P that minimize the function F.

Solutions:

1. 100 ft $\times$ 100 ft.

2. Base: 4 in $\times$ 4 in. Height: 4 in.

3. a) 50% b) 0% c) 50% d) 100%

4. Maximal concentration of .63 mg per cm^3 is present 2.24 hours after the medication is administered.

5. Base: 20 in $\times$ 20 in. Height: 15 in.

6. Dimensions 40 m $\times$ 10 m produce the minimal cost of \$ 320.

7. Base: 2 ft $\times$ 4 ft. Height: 3 ft

8. $P = 100$ kPa, $T = 50$ degrees Fahrenheit.