Math 102
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The Left and Right Sums

The Definite Integral: If f is a continuous function defined on interval $[a, b]$ then

$$\text{Area between the graph of } f \text{ and } x\text{-axis on } [a, b] = \int_a^b f(x)\ dx.$$

Approximating the area:

Step 1 Divide $[a, b]$ into n-pieces. Get the points $a = x_0,\ \ x_1,\ \ \ldots,\ \ x_{n-1},\ \ x_n = b$. This is called the **partition** of $[a, b]$. The distance between each two points is $\frac{b-a}{n}$.

Step 2 Approximate the area with the area of rectangles

$$\begin{aligned} \text{Left sum} &= \frac{b-a}{n}\ \ (f(x_0) + f(x_1) + \ldots + f(x_{n-1})) \\ \text{Right sum} &= \frac{b-a}{n}\ \ (f(x_1) + f(x_2) + \ldots + f(x_n\ )) \end{aligned}$$

The larger n, the better the approximation.

Practice Problems.

a) Approximate the integral with left and right sums using the partition with n subintervals.

1. $\int_0^2 x\ \ dx;\quad n = 2$
2. $\int_0^2 x^2\ \ dx;\quad n = 4$
3. $\int_0^3 8\sqrt{x}\ dx;\quad n = 100$ (use the calculator program for this)
4. $\int_1^4 x^{-2}\ dx;\quad n = 150$ (use the calculator program for this)

b) Approximate the following integral using your calculator program to first two nonzero digits.

1. $\int_0^2 \ln(x^2 + 1)\ dx$
2. $\int_1^3 \frac{e^{2x}}{x}\ dx$

c) The size of a certain bacteria culture grows at a rate of $f(t) = te^{t/2}$ milligrams per hour. Use your calculator program to approximate the total change in the bacteria size after the first 3 hours to first two nonzero digits.

Solutions.

a) 1. $L = 1, R = 3$ 2. $L = 1.75, R = 3.75$ 3. $L = 27.496, R = 27.912$ 4. $L = .759, R = .741$

b) 1. With $n = 100$, left sum = right sum =1.4 2. With $n = 300$, left sum = right sum = 81

c) With $n = 300$, get L=R=13 milligrams.