**MATH 102**
**Lia Vas**

# Review for Final Exam – Solutions

1. **Limits** a) $5/4$ b) $1$ c) $3$ d) $\infty$ e) $-\infty$ f) does not exist g) $0$ h) $2$ i) $0, 2,$ doesn't exist, $2, 0, 1.$ j) $0, 1,$ doesn't exist, $0, 4, 4.$

2. **Derivatives**

   a) $y' = -8/x^3 + 2/x^7$
   b) $y' = (1 + 1/x^2)(\sqrt[3]{x} + 5x^2) + (1/3x^{-2/3} + 10x)(x - \frac{1}{x})$
   c) $y' = \frac{(1/2x^{-1/2}+1)(x^2+1)-2x(\sqrt{x}+x)}{(x^2+1)^2}$
   d) $y' = \frac{[6x(x+4)+(3x^2+1)](5-x^2)+2x(3x^2+1)(x+4)}{(5-x^2)^2}$
   e) $y' = \frac{[1/2x^{-1/2}(2x^3-5)+\sqrt{x}6x](3x+2)-3\sqrt{x}(2x^3-5)}{(3x+2)^2}$
   f) $(5 - 5e^{5x})/(5x - e^{5x})$
   g) $(2x + 7)/(\ln 2(x^2 + 7x))$
   h) $(4(x^2 + 3)^3 2x(3x^2 + 1)^5 - 5(3x^2 + 1)^4 2x(x^2 + 3)^4)) \ / \ (3x^2 + 1)^{10}$
   i) $1/2 \left(\frac{x^2-1}{x^2+2}\right)^{-1/2} \frac{2x(x^2+2)-2x(x^2-1)}{(x^2+2)^2}$
   j) $3e^{3x}(x^3 + 2x - 5) + (3x^2 + 2)e^{3x}$
   k) $3^{2x^2+5} \cdot \ln 3 \cdot 4x$
   l) $5^{3x} + 5^{3x} \cdot \ln 5 \cdot 3x$
   m) $\cos(2x^2 + 4) \cdot (4x)$
   n) $2x \cos(x^2) - \sin(x^2) \cdot 2x \cdot x^2$
   o) $\cos 3x \cdot 3 \cdot \cos 5x - \sin 5x \cdot 5 \cdot \sin 3x$

3. **Implicit Differentiation**

   a) $dy/dx = -(2x + y)/x$
   b) $dy/dx = (x^2 + 4y)/(y^2 - 4x)$
   c) $dy/dx = (e^y + 2x)/(2y - xe^y)$

4. **Average and instantaneous rate of change**

   a) $3; 17.5$
   b) $-1; -3$

5. **Tangent Line**
   a) $y = 13/6x - 1/6$ b) $y = x$ c) $y = x - 1$ d) $y = -3/2x + 13/2$ e) $y = 2x - 1$ f) $y = 2x - 2$

6. **Linear Approximations**

   a) 5.3

   b) 4.7

   c) 2.962963. Calculator value: 2.962496.

   d) 2.00625. Calculator value: 2.00622.

7. **Applications of Derivatives**

   a) When production changes from 100 to 150 items produced, the cost increased at an average rate of \$32.5 per item produced. When producing 200 items, the cost is increasing at a rate of about \$31 per item produced. $C(201) \approx 7631$.

   b) $h(64) = 24$. 64 years after it starts growing, the tree is 24 feet tall. $h'(64) = 3/16 = .1875 \approx 0.19$. 64 years after it starts growing, the tree is growing at the rate of .19 feet per year.

   c) $N(9) = 8748$ mg, 9 minutes after, the size of bacteria is 8748 mg. $N'(9) = 3402$ mg per minutes, the population is increasing at the rate of 3402 milligrams per minute. $N(9.2) \approx 9428.4$ milligrams.

   d) The average velocity between 1 and 3 seconds after the stone is dropped is 64 feet per second. The velocity 2.5 seconds after the stone is dropped is 80 feet per second.

   e) Maximal concentration of 19.17% is present 3 hours after the medication is administered.

   f) Maximal revenue of \$373.29 is obtained when 67 items are sold.

8. **Related Rates** a) $1/(10\pi)$ cm per min. b) $3/(20\pi)$ cm per min. c) $-160$ bass per year d) $1/\pi$ in per min.

9. **Derivatives and Graphs**

   a) $f$ is increasing for $x < -5$ and $x > 3$; decreasing for $-5 < x < 3$ ; $f$ is concave upwards for $x > -1$; concave downward for $x < -1$; the relative minimum is (3, -24); the relative maximum is (-5, 61.33); the inflection point is (-1, 18.67).

   b) $f$ is increasing for $x < -4$ and $x > 4$ ; decreasing for $-4 < x < 0$ and $0 < x < 4$; $f$ is concave upwards for $x > 0$ ; concave downward for $x < 0$; the relative minimum is (4, 1/2); the relative maximum is (-4, -1/2); there are no inflection points.

   c) $f$ is increasing for $0 < x < e$ ; decreasing for $x > e$ ; $f$ is concave upwards for $x > e^{3/2}$; concave downward for $0 < x < e^{3/2}$; there is no relative minimum; the relative maximum is $(e, (1 + e)/e) = (2.72, 1.37)$; the inflection point is $(e^{3/2}, ) = (4.48, 1.33)$.

   d) $f$ is increasing for $x > -1/2$ ; decreasing for $x < -1/2$ ; $f$ is concave upwards for $x > -1$; concave downward for $x < -1$; the relative minimum is $(-1/2, -.18)$; there is no relative maximum; the inflection point is $(-1, -.135)$.

10. **Graphical Analysis**

    a) (0,1) is a relative minimum. $f$ is concave upwards for all values of $x$.

    b) (3,1) is a relative maximum, (-3, -1) is a relative minimum. $f$ is concave upward for $x < 0$, $f$ is concave downward for $x > 0$. (0,0) is an inflection point.

11. **Optimizing Functions**

    a) Absolute maximum (4, 449), absolute minimum (2, -63)

    b) Absolute maximum (1, 18), absolute minimum (3.2, -21)

12. **Optimization Problems**

    a) 100 ft $\times$ 100 ft.

    b) Base: 4 in $\times$ 4 in. Height: 4 in.

    c) a) 50% b) 0% c) 50% d) 100%

    d) Maximal concentration of .63 mg per $cm^3$ is present 2.24 hours after the medication is administered.

    e) Base: 20 in $\times$ 20 in. Height: 15 in.

13. **Definite and Indefinite Integrals.**

    a) $2/3x^{3/2} + 4/x + c$

    b) -9

    c) $1/21 \cdot (3x + 5)^7 + c$

    d) 52

    e) $\ln|x| + 1/x + c$

    f) $1/4 \ln|4x + 1| + c$

    g) $1/3 \ln|x^3 + 1| + c$

    h) $1/2e^{2x} - 1/2e^{-2x} + c$

    i) $4/\ln 3 = 3.64$

    j) $1/5 \sin(5x + 1) + c$

    k) 1

14. **Approximate Integration.**

    a) With $n = 100$, left sum = right sum =1.4

    b) With $n = 300$, left sum = right sum = 81

15. **Area.** a) 1.33 b) 2 c) 32/3 d) 2.61 e) 9/2 f) 1/2

16. **Applications of Integrals.**

a) With $n = 300$, get L=R=13 milligrams.

b) 4 ft

c) a) 1.59 ft b) $\sqrt{t^2 + 9} + 3$

d) $4192.834 \approx 4193$ thousands of barrels