Math 102
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Applications of Differentiation

1. A company determines that its cost function is $C(x) = 1000+35x-.01x^2$, $0 \leq x \leq 300$, where x is the number of items produced and $C(x)$ is the cost of producing x items in dollars. Find the average rate of change in cost when x is changing from 100 to 150. Then, find the instantaneous rate of change in cost when producing 200 units and estimate the cost of producing 201 items.

2. Assume that the mathematical model for the growth of a locust tree in its first century of life is given by $h(t) = 3\sqrt{t}$, $0 \leq t \leq 100$, where t is the age of the tree in years and $h(t)$ is the height of the tree in feet. Find and interpret $h(64)$ and $h'(64)$.

3. The mass of bacteria culture at time t in hours, is approximated by $N(t) = 4t^{7/2}$, in milligrams. Find and interpret $N(9)$ and $N'(9)$. How fast is the mass of bacteria increasing 4 hours after the experiment started?

4. The mass of a bacteria culture t hours after the start of experiment, is modeled by $N(t) = 3t^{5/2}$, in milligrams. a) What is the mass 16 hours after experiment started? b) How fast is the mass of bacteria increasing 9 hours after the experiment started?

5. If a stone is dropped from a building 150 feet tall, its height above the ground after t seconds is given by $s(t) = 150 - 16t^2$, in feet. Find the average velocity of the stone between 1 and 3 seconds after it is dropped. Find the velocity 2.5 seconds after the stone is dropped.

6. A particle moves on a line away from its initial position so that after t hours it is $s(t) = 2t^2 - 1$ miles from its initial position. Find the average velocity of the particle between 2 and 4 hours after it started moving. Find the velocity of the particle 5 hours after it started moving.

7. The body mass index (BMI) is a number obtained as $BMI = \frac{703w}{h^2}$ where w is the weight in pounds and h is the height in inches. For a 125-lb female that is now 65 inches tall but growing, calculate how fast is BMI changing with each new inch.

8. The concentration of a certain medication in a patient's bloodstream (in mg per cm^3) is given by $C(t) = \frac{5t}{t^2+4}$, where t is the number of hours after the medication has been administered.
   a) What is the concentration 3 hours after the medication is administered?
   b) How fast is the concentration changing 3 hours after the medication is administered?
   c) How fast is the concentration changing on average between 2nd and 4th hour?

9. The profit P of a company depends on the number of items x produced. The production level x depends on the time t (measured in years). Assume that the profit increases by \$200 with each new item produced and that the production level increases by 150 items each year.

a) Write the sentence "the profit increases by \$200 with each new item produced" as a formula using the derivative notation. Do the same for the sentence "the production level increases by 150 items each year. "

b) How much is the profit increasing each year?

c) If the company is presently making a profit of \$800,000, find the profit in four year time.

10. The concentration of pollutants (in grams per liter) in a river is approximated by $P(x) = .04e^{-4x}$ where x is the number of miles downstream from a place where the measurements are taken.

    a) What was the initial pollution? What is the pollution 2 miles downstream?

    b) How much did the concentration changed on average within the first two miles?

    c) How fast is the concentration changing 2 miles downstream?

Solutions:

1. When production changes from 100 to 150 items produced, the cost increased at an average rate of \$32.5 per item produced. When producing 200 items, the cost is increasing at a rate of about \$31 per item produced. $C(201) \approx 7631$.

2. $h(64) = 24$. 64 years after it starts growing, the tree is 24 feet tall. $h'(64) = 3/16 = .1875 \approx 0.19$. 64 years after it starts growing, the tree is growing at the rate of .19 feet per year.

3. $N(9) = 8748$ mg = the mass of bacteria 9 hours after. $N'(9) = 3402$ mg per hour = the rate at which the mass is increasing after 9 hours. 4 hours after, the mass of bacteria is increasing at the rate of 448 mg per hour.

4. a) 3072 mg. b) 202.5 mg per hour.

5. The average velocity between 1 and 3 seconds after the stone is dropped is 64 feet per second. The velocity 2.5 seconds after the stone is dropped is 80 feet per second.

6. The average velocity from 2 to 4 hours is 12 miles per hour. Velocity 5 hours after is 20 miles per hour.

7. The value of the derivative of $\frac{703(125)}{h^2}$ at $h = 65$ is $-.6399 \approx -.64$. Thus, the BMI is decreasing by .64 per inch.

8. a) 1.15 mg/cm^3 b) $C'(3) = -.148$ thus, the concentration is decreasing by .148 mg/cm^3 per hour. c) $\frac{1-1.25}{4-2} = -.124$, thus the concentration is decreasing on average by .124 mg/cm^3 per hour between hour 2 and 4.

9. a) $\frac{dP}{dx} = 200$ dollars per item and $\frac{dx}{dt} = 150$ items per year. b) $\frac{dP}{dt} = \frac{dP}{dx}\frac{dx}{dt} = 200 \cdot 150 = 30,000$ dollars per year. c) $800,000 + 4 \cdot 30,000 = 920,000$ dollars.

10. a) $C(0) = .04$ and $C(2) = 1.3 \cdot 10^{-5}$ grams per liter b) $-.01999 \approx -.02$. Thus the concentration is decreasing on average by .02 grams per liter per mile during the first two miles. c) $C'(2) = -5.37 \cdot 10^{-5}$, thus the concentration is decreasing by .0000537 grams per liter per mile 2 miles downstream.