

## Trigonometric Functions

Basic trigonometric functions:

$$\sin x = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \cos x = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Connection:  $\sin^2 x + \cos^2 x = 1$

Other trigonometric functions:

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x}$$

### Practice problems:

1. Determine sine, cosine and tangent of an angle formed by the ray  $OP$ , where  $O$  is the origin and  $P$  is the point  $(5, -12)$ , and the positive part of  $x$ -axis.
2. Determine sine, cosine and tangent of an angle formed by the ray  $OP$ , where  $O$  is the origin and  $P$  is the point  $(-3, -4)$ , and the positive part of  $x$ -axis.
3. Sketch the right triangle with  $\tan \alpha = 4$ . Find all sides of the triangle,  $\sin \alpha$  and  $\cos \alpha$ .
4. Sketch the right triangle with  $\sin \alpha = 3/5$ . Find all sides of the triangle,  $\cos \alpha$  and  $\tan \alpha$ .
5. If  $\sin \alpha = -7/8$ , and  $\cos \alpha > 0$ , find other five trigonometric functions of  $\alpha$ .
6. If  $\cos \alpha = -5/11$ , and  $\sin \alpha > 0$ , find other five trigonometric functions of  $\alpha$ .
7. Evaluate the following:  $\sec(\pi/3)$ ,  $\cot(120^\circ)$ ,  $\sin^{-1}(1)$ ,  $\cos^{-1}(1/2)$ .
8. Solve  $\sin x = 2/5$
9. Solve  $\cos^2 x = 1/4$
10. Solve  $2 \tan x + 3 = 9$
11. Solve  $2 \sin^2 x = \cos x + 1$ .
12. Solve  $\sin x \cos x = \sin x$ .
13. Solve  $4 \sin^3 x = \sin x$ .
14. Solve  $\tan^3 x = \tan x$

15. Solve  $2 \sin^2 x = 1 - \cos x$
16. Solve  $2 \tan x \sin x = \tan x$
17. Find the inverse function of  $f(x) = 5 \sin \frac{x}{2} \quad -\pi < x \leq \pi$ .
18. Find the inverse function of  $f(x) = 3 \tan(2x + \pi)$ .
19. Find the composite functions  $f \circ g$  and  $g \circ f$  if  $f(x) = \sin x$  and  $g(x) = x^2$ .
20. The size of a population of rabbits changes periodically because of seasonal changes. The number of rabbits depending on the time  $t$  (in months within a year) can be described by the formula  $N(t) = 100 - 20 \cos(t \cdot 30^\circ)$ . At what time(s) is there exactly 110 rabbits?
21. A function model for the average temperature in Philadelphia during a year is

$$T(x) = 20 \sin\left(\frac{2\pi}{365}(x - 91)\right) + 16$$

- where  $T$  is the temperature in degrees Celsius on day  $x$ . The day  $x = 0$  corresponds to January 1st and  $x = 364$  to December 31st. Use this model to estimate the temperature on February 1st. Find the day(s) when the temperature is 26 degrees Celsius.
22. An observer in a lighthouse 327 feet above the sea level spots two boats. The angle of depression of the boats are  $13.42^\circ$  and  $16.16^\circ$ . Find the distance between the boats.
  23. Standing 300 feet from City Hall, you estimate that the angle of elevation from the ground to the base of the William Penn statue is 59.6 degrees, and that the angle of elevation from the ground to the top of the William Penn's hat is 61.3 degrees. Estimate the height of the statue.
  24. Two buildings A and B are 50 feet apart. The angle of elevation from the bottom of A to the top of B is 45 degrees. The angle of depression from the top of B to the top of A is 30 degrees. How tall is building A?
  25. The table below shows the average temperature (in degrees) in a certain city over the course of a year.

month	<i>J</i>	<i>F</i>	<i>M</i>	<i>A</i>	<i>M</i>	<i>J</i>	<i>J</i>	<i>A</i>	<i>S</i>	<i>O</i>	<i>N</i>	<i>D</i>
avg. temp.	31.2	34.1	42.6	46.3	57.6	66.8	77.9	74.5	68.3	57.7	44.3	38.4

Find the sine function to best fit the data. Predict the average temperature in March the following year.

**Solutions:**

1.  $\sin \alpha = -12/13$ ,  $\cos \alpha = 5/13$ ,  $\tan \alpha = -12/5$

2.  $\sin \alpha = -4/5$ ,  $\cos \alpha = -3/5$ ,  $\tan \alpha = 4/3$
3. Opposite = 4, adjacent = 1, hypotenuse =  $\sqrt{17}$ ,  $\sin \alpha = 4/\sqrt{17}$ ,  $\cos \alpha = 1/\sqrt{17}$ .
4. Opposite = 3, hypotenuse = 5, adjacent = 4,  $\cos \alpha = 4/5$ ,  $\tan \alpha = 3/5$ .
5.  $\cos \alpha = \sqrt{15}/8$ ,  $\tan \alpha = -7/\sqrt{15}$ ,  $\cot \alpha = -\sqrt{15}/7$ ,  $\sec \alpha = 8/\sqrt{15}$ ,  $\csc \alpha = -8/7$ .
6.  $\sin \alpha = \sqrt{96}/11$ ,  $\tan \alpha = -\sqrt{96}/5$ ,  $\cot \alpha = -5/\sqrt{96}$ ,  $\sec \alpha = -11/5$ ,  $\csc \alpha = 11/\sqrt{96}$ .
7.  $2$ ,  $-1/\sqrt{3} = -.557$ ,  $\pi/2 = 90$  degrees,  $\pi/3 = 60$  degrees.
8.  $x = 23.57$  degrees and  $156.42$  degrees or  $x = .411$  and  $2.73$  radians.
9.  $x = 60, 120, 240, 300$  or  $x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$  radians.
10.  $x = 71.56, 251.57$  degrees or  $x = 1.25, 4.39$  radians.
11.  $x = 60, 180, 300$  degrees or  $x = \pi/3, \pi, 5\pi/3$  radians.
12.  $x = 0, 180$  degrees or  $x = 0, \pi$  radians.
13.  $x = 0, 30, 150, 180, 210, 330$  degrees or  $x = 0, \pi/6, 5\pi/6, \pi, 7\pi/6, 11\pi/6$  radians.
14.  $x = 0, 45, 135, 180, 225, 315$  degrees or  $x = 0, \pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4$  radians.
15.  $x = 0, 120, 240$  degrees or  $x = 0, 2\pi/3, 4\pi/3$  radians.
16.  $x = 0, 30, 150, 180$  degrees or  $x = 0, \pi/6, 5\pi/6, \pi$  radians.
17.  $f^{-1}(x) = 2 \sin^{-1} \frac{x}{5}$
18.  $f^{-1} = 1/2(\tan^{-1}(x/3) - \pi)$
19.  $(f \circ g)(x) = \sin(x^2)$  and  $(g \circ f)(x) = (\sin x)^2$
20.  $t = 4$  and  $t = 8$ , so beginning of May and beginning of September.
21. February 1st = day 31.  $T(31) = -1$  degrees Celsius.  $T(x) = 26$  when  $x = 121$  and  $x = 243$ .
22. 242 feet.
23. 36.6 feet.
24.  $50 - 50/\sqrt{3} = 21.13$  feet.
25. 40.3 degrees.