MATH 101
Lia Vas

Review 3 – Solutions

1. Domains.
   a) all real numbers b) $x > 2$ and $x \neq 3$ c) $x < -2$ or $x > 2$ and $x \neq 3$ d) $x > 2$ and $x \neq 3$ e) $1 < x < 5$ f) $x > 5$ g) $1 < x \leq 5$ h) $x < -5$ or $x > 3$ i) $-2 < x < 0$ or $x > 4$ j) $x > 3$ or $x < -3$ k) $x \geq 3,\ x \leq -3$ or $x \neq 5$ l) $x \leq -5$ or $x \geq 3$ m) $-2 \leq x \leq 0$ or $x \geq 4$

2. Power Functions.
   a) $\ln y = 1.5 + 3\ln x$ means that $y = e^{1.5}x^3 = 4.48x^3$ b) $\ln y = -.5 - 2\ln x$ means that $y = e^{-.5}x^{-2} = \frac{.607}{x^2}$ c) $y = 5\sqrt{x}$ implies that $\ln y = \ln 5 + \frac{1}{2}\ln x = 1.61 + \frac{1}{2}\ln x$ d) $y = 2.23x^{4.54}$

3. Polynomials.
   a) Second-order difference is constant (4) so the data are quadratic. Quadratic function that fits the data is $y = 2x^2 - 3x + 1$. b) Degree 4, leading coefficient positive. c) Degree 5, leading coefficient negative. d) In .8 hours and 2.4 hours.

   e) Part a) Use quadratic or cubic model. The leading coefficient of quartic model is $-9.7 \cdot 10^{-6}$. As this number is very small, this means that this curve is almost a cubic. Thus, cubic is more efficient and equally reliable. Part b) Using quadratic: 195 units cost \$1287. c) Using quadratic: about 42 units. Using cubic: about 41 units.

   f) Quartic is better. Maximum 19.5 days after. No infected students 33 days after.

   g) The cubic model is a slightly better fit than quadratic. For cubic $R^2 = .9999$. The quartic model has almost the same R^2 as cubic so it increases complexity without adding much accuracy. Using cubic model, the population after 7 days is 1886 bacteria.

4. Rational Functions.
   a) Horizontal $y = 1$, vertical (and pole) $x = -1$.
   b) Horizontal $y = 0$, vertical (and poles) $x = 1$, and $x = -3$.
   c) No horizontal, vertical and pole $x = 2$.
   d) Horizontal $y = 2/3$, vertical (and poles) $x = 1$ and $x = -2/3$.
   e) Horizontal $y = 0$, $x = 0$ is a pole but not a vertical asymptote. $x = 4$ is both a pole and a vertical asymptote.
   f) No horizontal and vertical asymptotes. $x = 1$ is a pole.

5. Piecewise functions.
   a) $f(-2) = -1$, $f(1) = 2$, $f(2) = 1$.
   b) $f(-2) = 0$, $f(1) = 1$, $f(3) = -1$.

c) $f(-3) = -1$, $f(1) = 0$, $f(3) = 1$.

d) a) $y = 5 + 2x$. b) $y = \begin{cases} 5 + 2x & x \leq 3 \\ 6.5 + 1.5x & x > 3 \end{cases}$ c) 9 months.

e) $y = \begin{cases} 3 + .125x & x \leq 2 \\ 3.55 - .15x & x > 2 \end{cases}$ The concentration will drop below 2 μg/ cm^3 10.333 hours (or 10 hours and 20 minutes) after the medication is given.

f) Before 5 hours, the exponential function has initial size 50 and the growth rate 2, so it is $y = 50(2)^t$ for $t \leq 5$. When $t = 5$, $y = 1600$. After 5 hours, the function is changing to a line with slope 60, passing (5, 1600). So, it is $y = 60t + 1300$ for $t > 5$. Thus, the piecewise function is $y = \begin{cases} 50(2)^t & t \leq 5 \\ 60t + 1300 & t > 5 \end{cases}$ $y(4) = 800$ mg and $y(6) = 1660$ mg.

g) The exponential regression for the first three given points gives us the formula $y = 29.96(1.5055)^x$. The exponential regression for the next four points gives us $y = 14.56(1.6028)^x$. Thus, the piecewise function is $y = \begin{cases} 29.96(1.5055)^x & 0 \leq x \leq 2 \\ 14.56(1.6028)^x & x \geq 3 \end{cases}$ $y(10) = 1629.47 \approx$ 1629 rabbits.

6. Hardy-Weinberg Theorem.
a) .6 b) 32% c) Heterozygous 48%. Homozygous 52%. d) 96% display dominant phenotype. 32% display dominant phenotype and carry recessive allele. 4% display recessive phenotype. e) 49%

7. (a) In 1986. (b) 9.36 hours (approximately 9 hours and 20 minutes) (c) 172 prescriptions. In year 2021. (d) Logarithmic model. 10.34 min. (e) Exponential model $y = 247.809(1.4297)^x$. 3025 bacteria. 8.4 hours (8 hours 24 minutes).