MATH 101
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Review for Final Exam – Solutions

1. Trigonometric Equations.

   a) x =23.57 degrees and 156.42 degrees or $x = .411$ and 2.73 radians.

   b) $x = 60, 120, 240, 300$ or $x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$ radians.

   c) $x = 71.56, 251.57$ degrees or $x = 1.25, 4.39$ radians.

   d) $x = 0, 180$ degrees or $x = 0, \pi$ radians.

   e) $x = 0, 30, 150, 180, 210, 330$ degrees or $x = 0, \pi/6, 5\pi/6, \pi, 7\pi/6, 11\pi/6$ radians.

   f) $x = 0, 45, 135, 180, 225, 315$ degrees or $x = 0, \pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4$ radians.

   g) $x = 0, 120, 240$ degrees or $x = 0, 2\pi/3, 4\pi/3$ radians.

2. Inverse Trigonometric Functions.
   a) $f^{-1}(x) = 2\sin^{-1}\frac{x}{5}$ b) $f^{-1}(x) = 1/2(\tan^{-1}(x/3) - \pi)$

3. Right Angle Trigonometry. Trigonometric Applications.

   a) $\sin\alpha = -12/13$, $\cos\alpha = 5/13$, $\tan\alpha = -12/5$

   b) $\sin\alpha = -4/5$, $\cos\alpha = -3/5$, $\tan\alpha = 4/3$

   c) Opposite = 4, adjacent =1, hypotenuse = $\sqrt{17}$, $\sin\alpha = 4/\sqrt{17}$, $\cos\alpha = 1/\sqrt{17}$.

   d) Opposite = 3, hypotenuse = 5, adjacent =4, $\cos\alpha = 4/5$, $\tan\alpha = 3/5$.

   e) $\cos\alpha = \sqrt{15}/8$, $\tan\alpha = -7/\sqrt{15}$, $\cot\alpha = -\sqrt{15}/7$, $\sec\alpha = 8/\sqrt{15}$, $\csc\alpha = -8/7$.

   f) $\sin\alpha = \sqrt{96}/11$, $\tan\alpha = -\sqrt{96}/5$, $\cot\alpha = -5/\sqrt{96}$, $\sec\alpha = -11/5$, $\csc\alpha = 11/\sqrt{96}$.

   g) $t = 4$ and $t = 8$, so beginning of May and beginning of September.

   h) 242 feet.

   i) 36.6 feet.

   j) $50 - 50/\sqrt{3} = 21.13$ feet.

   k) 40.3 degrees

4. Conversions. a) $8.88 \cdot 10^8$ in^2 b) $3.26 \cdot 10^1$ m^3 c) $6.69 \cdot 10^{23}$ molecules d) $2.99 \cdot 10^{-3}$ grams

5. Factoring. a) $x^6y^{-6}(5y - 3x)$ b) $x^4(4x - 3y)(4x + 3y)$ c) $-(x + 2)(x - 5)$ d) $(3x + 1)(2x - 1)$ e) $x^{-3}(2x + 1)(4x - 3)$ f) $x^{-2}(x + 4)(x - 2)$

6. Equations.

   (a) $x = \pm 4$

(b) $x = 2$

(c) $x = 12$ and $x = 4$

(d) $x = -.229$

(e) $x = 60, 120, 240, 300$ or $x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$ radians.

(f) $x = 0, 180$ degrees or $x = 0, \pi$ radians.

(g) $x = 0, 30, 150, 180, 210, 330$ degrees or $x = 0, \pi/6, 5\pi/6, \pi, 7\pi/6, 11\pi/6$ radians.

7. Line. a) $y = -2/5x - 19/5$ b) $y = 5/2x - 25/2$ c) $y = -7/2x - 49/2$

8. Domain and Range. a) Domain: $\mathcal{R}$, range: $\mathcal{R}$ b) Domain: $\mathcal{R}$, range: $y \geq 2$ c) Domain: $x \geq 2$, range: $y \geq 0$ d) Domain: $x \geq 2$, range: $y \leq 1$ e) Domain: $x \neq 2$, range: $y =\neq 0$

9. Piecewise Functions. a) $f(-2) = 0$, $f(1) = 1$, $f(3) = -1$. b) $f(-3) = -1$, $f(1) = 0$, $f(3) = 1$.

   c) $y = \begin{cases} 3 + .125x & x \leq 2 \\ 3.55 - .15x & x > 2 \end{cases}$ The concentration will drop below 2 μg/ cm^3 10.333 hours (or 10 hours and 20 minutes) after the medication is given.

   d) Before 5 hours, the exponential function has initial size 50 and the growth rate 2, so it is $y = 50(2)^t$ for $t \leq 5$. When $t = 5$, $y = 1600$. After 5 hours, the function is changing to a line with slope 60, passing (5, 1600). So, it is $y = 60t + 1300$ for $t > 5$. Thus, the piecewise function is $y = \begin{cases} 50(2)^t & t \leq 5 \\ 60t + 1300 & t > 5 \end{cases}$ $y(4) = 800$ mg and $y(6) = 1660$ mg.

   e) The exponential regression for the first three given points gives us the formula $y = 29.96(1.5055)^x$. The exponential regression for the next four points is $y = 14.56(1.6028)^x$. Thus, the piecewise function is $y = \begin{cases} 29.96(1.5055)^x & 0 \leq x \leq 2 \\ 14.56(1.6028)^x & x \geq 3 \end{cases}$ $y(10) = 1629.47 \approx 1629$ rabbits.

10. Hardy-Weinberg. a) 48% b) 36% c) .6

11. Functions by tables.

   a) a) Average rate = .67. This means that the size of bacteria is increasing by .67 mg every hour between 20 and 30 hours after the experiment started. b) 48.27 mg c) Average rate = .19. This means that the size of bacteria is increasing by .19 mg every hour between 30 and 40 hours after the experiment started. d) 52.63 mg e) 54 mg.

   b) a) From table $y = 1.49$. So, 1.49 grams. b) $\frac{.81-1.49}{15-10} = -.136$ grams per thousand years. Thus, the amount is decreasing by .136 grams every 1000 years. c) $1.49 - .136(12 - 10) = 1.218$ grams d) 0 grams.

12. Systems.

   a) One solution $\{(0, -1, 2, 1)\}$.

b) No solution.

c) Infinitely many solutions. Set of solutions: $\{(-1, 4-2z, z) \mid z \text{ any real number}\}$.

d) Infinitely many solutions. Set of solutions: $\{(-26+3z, 11-z, z) \mid z \text{ any real number}\}$.

e) 2 cups of milk, 3 cups of orange juice and $1\frac{1}{2}$cups of tomato juice.

13. Domains. a) $\{x \mid x \geq 2 \text{ or } x \neq 3\}$ b) $\{x \mid x \geq 3,\ x \leq -3 \text{ or } x \neq 5\}$ c) $\{x \mid x \leq -5 \text{ or } x \geq 3\}$ d) $\{x \mid -2 \leq x \leq 0 \text{ or } x \geq 4\}$ e) $\{x \mid x < -2 \text{ or } x > 2 \text{ and } x \neq 3\}$ f) $\{x \mid x > 2 \text{ and } x \neq 3\}$ g) $\{x \mid x < -5 \text{ or } x > 3\}$ h) $\{x \mid -2 < x < 0 \text{ or } x > 4\}$

14. Inverse Function.

a) $f^{-1}(x) = (x-5)^3 - 1$
b) $f^{-1}(x) = \frac{4+5x}{3-2x}$ (same as $\frac{-4-5x}{2x-3}$)
c) $f^{-1}(x) = \frac{x}{2-x}$ (same as $\frac{-x}{x-2}$)
d) $f^{-1}(x) = 1/3(\log_2 x + 1)$
e) $f^{-1}(x) = \frac{3^{x-4}}{2}$
f) $f^{-1}(x) = 5^x + 3$
g) $f^{-1}(x) = 2\sin^{-1}\frac{x}{5}$
h) $f^{-1} = 1/2(\tan^{-1}(x/3) - \pi)$

15. Composite. Intercepts.

a) $(f \circ g)(x) = \sqrt{\ln(x-1)+2}$, $(g \circ f)(x) = \ln(\sqrt{x+2} - 1)$. No y-intercept of $g(x)$, x-intercept=(2,0).

b) $(f \circ g)(x) = (\ln(x) - 2)^3$, $(g \circ f)(x) = \ln(x^3) - 2$. No y-intercept of $g(x)$, x-intercept $= (e^2, 0) = (7.39, 0)$.

c) $(g \circ f)(x) = (\log_2(x+3))^2$, $(f \circ g)(x) = \log_2(x^2+3)$. y-intercept of $g(x) = (0, \log_2(3)) = (0, 1.58)$. x-intercept = (-2,0).

16. Rational Functions.

a) Horizontal $y = 1$, vertical (and pole) $x = -1$.

b) Horizontal $y = 0$, vertical (and poles) $x = 1$, and $x = -3$.

c) No horizontal, vertical and pole $x = 2$.

d) Horizontal $y = 2/3$, vertical (and poles) $x = 1$ and $x = -2/3$.

e) Horizontal $y = 0$, $x = 0$ is a pole but not a vertical asymptote. $x = 4$ is both a pole and a vertical asymptote.

17. Linear Modeling. a) 160 cm^3 b) 75 liters c) 9 liters of 80% solution and 3 liters of 60% solution d)$y = -3x + 110$. 35 days. 6 degrees. e) $y = -2x + 90$. 22 days. 30 degrees. f) $y_1 = 10 + 2x$, $y_2 = 120 + .75x$. Intersection $x = 88$, so when used 88 times, they cost the same. If used more than 88 times, it is cheaper to buy more sophisticated thermometer. g) $y_1 = 40,000 + 20x$, $y_2 = 32,000 + 30x$. Intersection $x = 800$, so when used 800 times, they cost the same. If used more than 800 times, it is cheaper to buy a machine from Acme Products.

18. Exponential Growth and Decay. a) 200 bacteria. 3118 bacteria. b) 11.6 days c) $P = P_0(1.05)^t$ or $P_0e^{.05t}$. $t = 22$ years.

    d) a) Smileytown; b) $y_1 = 10000 + 1500t, y_2 = 10000(1.15)^t$; or $10000e^{.15t}$ c) 17500 and 20114 people; d) $t = 10$ years and $t = 6.556 \approx 6.5$ years.

19. Other Function Applications.

    a) 419.74 mg/person. Min. value at $x = 18$ approx. which corresponds to year 1992.
    b) Maximum height is 16 feet. It will hit the ground after 2 sec.
    c) 220 grams. Approx. 42 cm
    d) $6.31 \cdot 10^{-6}$ moles per liter. pH is 7.4.
    e) Initially, 1 student; week later, 238; 9.25 days after, 45% of 2000 are infected.

20. Regressions.

    a) Negative slope of -26.64 means that the area of the wound is decreasing at the rate of 26.64 mm^2 per day. The would will be healed after 7.5 days.
    b) Quadratic. 195 units cost $1287. 42 units for $800.
    c) Quartic is better. Maximum 19.5 days after. No infected students 33 days after.
    d) 172 prescriptions. In year 2021.
    e) Logarithmic model. $y = 1.12 + 8.08 \ln x$, $R^2 = .9953$. In 10.34 minutes the yield will be 20 mg. The quartic model has a slightly larger R^2 but starts decreasing after 6 minutes.