MATH 101
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Review for Final Exam

1. **Trigonometric Equations.** Solve for x. Express your answer both in degrees ($0 \leq x < 360$) and in radians ($0 \leq x < 2\pi$).

   a) $\sin x = 2/5$
   b) $\cos^2 x = 1/4$
   c) $2\tan x + 3 = 9$
   d) $\sin x \cos x = \sin x$.
   e) $4\sin^3 x = \sin x$.
   f) $\tan^3 x = \tan x$
   g) $2\sin^2 x = 1 - \cos x$

2. **Inverse Trigonometric Functions.** For given function f find the inverse function f^{-1}.

   a) $f(x) = 5\sin\frac{x}{2}$ b) $f(x) = 3\tan(2x + \pi)$.

3. **Right Angle Trigonometry. Trigonometric Applications.**

   a) Determine sine, cosine and tangent of an angle formed by the ray OP, where O is the origin and P is the point $(5, -12)$, and the positive part of x-axis.
   b) Determine sine, cosine and tangent of an angle formed by the ray OP, where O is the origin and P is the point $(-3, -4)$, and the positive part of x-axis.
   c) Sketch the right triangle with $\tan\alpha = 4$. Find all sides of the triangle, $\sin\alpha$ and $\cos\alpha$.
   d) Sketch the right triangle with $\sin\alpha = 3/5$. Find all sides of the triangle, $\cos\alpha$ and $\tan\alpha$.
   e) If $\sin\alpha = -7/8$, and $\cos\alpha > 0$, find other five trigonometric functions of α.
   f) If $\cos\alpha = -5/11$, and $\sin\alpha > 0$, find other five trigonometric functions of α.
   g) The size of a population of rabbits changes periodically because of seasonal changes. The number of rabbits depending on the time t (in months within a year) can be described by the formula $N(t) = 100 - 20\cos(t\pi/6)$. At what time(s) is there exactly 110 rabbits?
   h) An observer in a lighthouse 327 feet above the sea level spots two boats. The angle of depression of the boats are $13.42°$ and $16.16°$. Find the distance between the boats.
   i) Standing 300 feet from City Hall, you estimate that the angle of elevation from the ground to the base of the William Penn statue is 59.6 degrees, and that the angle of elevation from the ground to the top of the William Penn's hat is 61.3 degrees. Estimate the height of the statue.

j) Two buildings A and B are 50 feet apart. The angle of elevation from the bottom of A to the top of B is 45 degrees. The angle of depression from the top of B to the top of A is 30 degrees. How tall is building A?

k) The table below shows the average temperature (in degrees) in a certain city over the course of a year.

month	J	F	M	A	M	J	J	A	S	O	N	D
avg. temp.	31.2	34.1	42.6	46.3	57.6	66.8	77.9	74.5	68.3	57.7	44.3	38.4

Find the sine function to best fit the data. Predict the average temperature in March the following year.

4. **Conversions.** Convert:

a) $5.73 \cdot 10^5$ square meters into square inches using that 1 inch = 2.54 cm and 1 cm = 10^{-2} m.

b) $3.26 \cdot 10^4$ liters to cubic meters using that 1 l = 10^3 cubic centimeters and 1 cm = 10^{-2} m.

c) 20 g of water to number of molecules using that the atomic weights of hydrogen and oxygen are 1 and 16 and the Avogadro's number (number of molecules in a mole) $6.02 \cdot 10^{23}$.

d) Find how much do 10^{20} molecules of water weigh using that the atomic weights of hydrogen and oxygen are 1 and 16 and the Avogadro's number (number of molecules in a mole) $6.02 \cdot 10^{23}$.

5. **Factoring.** Factor the given expression leaving all exponents positive:

a) $5x^6y^{-5} - 3x^7y^{-6}$

b) $16x^6 - 9x^4y^2$

c) $-x^2 + 3x + 10$

d) $6x^2 - x - 1$

e) $8x^{-1} - 2x^{-2} - 3x^{-3}$

f) $1 + 2x^{-1} - 8x^{-2}$

6. **Equations.** Solve for x. For trigonometric equations, express your answer both in degrees ($0 \leq x < 360$) and in radians ($0 \leq x < 2\pi$).

(a) $\log_5(x^2 + 9) = 2$

(b) $e^{4x-8} = 1$

(c) $2\ln x - 2\ln 4 = \ln(x - 3)$

(d) $3^{x+2} = 7$

(e) $\cos^2 x = 1/4$

(f) $\sin x \cos x = \sin x$.

(g) $4\sin^3 x = \sin x$.

7. **Line**. Find the equation of the line that is:

   a) parallel to $2x + 5y = 11$ and passing the point (3,-5);

   b) perpendicular to $2x + 5y = 11$ and passing the point (3,-5);

   c) perpendicular to $2x - 7y = 11$ and has -7 as x-intercept.

8. **Domain and Range.** Find the domain and range of the following functions:

   a) $y = x^3 - 2$

   b) $y = x^4 + 2$

   c) $y = \sqrt{x - 2}$

   d) $y = 1 - \sqrt{x - 2}$

   e) $y = \frac{3}{2x-4}$

9. **Piecewise Functions**.

   a) Graph $y = \begin{cases} 2 - x & x \geq 1 \\ x & -1 \leq x < 1 \\ -x - 2 & x < -1 \end{cases}$ Find $f(-2)$, $f(1)$, $f(3)$.

   b) Graph $y = \begin{cases} x + 2 & x < -2 \\ 0 & -2 \leq x < 2 \\ (x - 2)^2 & x \geq 2 \end{cases}$ Find $f(-3)$, $f(1)$, $f(3)$.

   c) The concentration of a medication in patients body increases linearly during the first two hours. Initially, it is 3 μg/ cm^3, and after two hours it is 3.25 μg/ cm^3. After two hours, the concentration starts decreasing so that 5 hour after, it is 2.80 μg/ cm^3. Write a piecewise linear function that describes the concentration of medication as a function of number of hours passed. Sketch this function. When will the concentration drop below 2 μg/ cm^3?

   d) Initially there is 50 mg of bacteria culture. The number of bacteria is doubling every hour for the first five hours. What is the bacteria size after 5 hours? After 5 hours, the growth rate slows down and the culture increases by 60 mg every hour. Write down a piecewise function that describes the bacteria size as function of time in hours. Sketch the function. Use the formula to find the bacteria size 4 and 6 hours after the start of experiment.

   e) The size of a population of rabbits in a certain habitat is described by a table below.

year	2000	2001	2002	2003	2004	2005	2006
number of rabbits	30	45	68	60	96	154	247

   We can see that the number is increasing from 2000 to 2003. There was a decrease in number of rabbits in 2004 due to a flood in the habitat but after 2004, the number of rabbits is increasing again. Assuming that the number of rabbits is increasing exponentially both before the flood and after the flood, find the two exponential regressions that will best fit the data before and after the flood. Using the two formulas, write down a piecewise function that will describe the number of rabbits from 2000 to 2006. Estimate the number of rabbits in 2010.

10. **Hardy-Weinberg**.

a) In a certain wild flower population we find yellow and white flowers. The allele for yellow color is dominant. If 16% of this population are white flowers, find the percentage of flowers that are yellow and carry the recessive allele in their chromosomes.

b) Within a population of butterflies the color brown is dominant over the color white. If 84% of all butterflies are brown, find the percentage of butterflies with just dominant alleles in their chromosomes.

c) If 84% of the population displays a dominant trait, what is the frequency of dominant allele?

11. **Functions by Tables.**

a) In an experiment, a bacteria culture is monitored and its size (in mg) is being recorded every 10 hours. Let

time (hours)	0	10	20	30	40	50	60
size (mg)	17.6	23.8	44.6	51.3	53.2	53.7	53.9

a) What is the average rate of change from 20 to 30 hours? Explain what that answer means. b) Use your answer to estimate the size 23 hours after the start of experiment. c) What is the average rate of change from 30 to 40? d) Use your answer to estimate the size 37 hours after the start of experiment. e) What is the expected size of this bacteria culture on the long run? (i.e. what is the limiting value of the size?)

b) Carbon 14 is a radioactive substance that decays over time. In the following table, time t is measured in thousands of years and C is the amount in grams of carbon 14 remaining.

time (thousands of years)	0	5	10	15	20
amount remaining (grams)	5	2.73	1.49	0.81	0.44

a) What quantity of carbon 14 will be present after 10,000 years? b) What is the average rate of change of the amount of carbon 14 between 10,000 and 15,000 years? Explain what the sign of your answer means. Include the units. c) Use the answer from b) to estimate the amount of carbon 14 after 12,000 years. d) How many grams of carbon 14 do you expect to find after a very large number of thousands of years?

12. **Systems of equations.** Solve the following systems of equations:

a)

$$\begin{array}{rrrrcr} 5x & +2y & -2z & +3u & = & -3 \\ 2x & +y & -z & -2u & = & -5 \\ -3x & +4y & -2z & +2u & = & -6 \\ -4x & -2y & +3z & +u & = & 9 \end{array}$$

b)

$$\begin{array}{rrrcr} x & -3y & -6z & = & 0 \\ -3x & -2y & +7z & = & 4 \\ 2x & +5y & -z & = & 3 \end{array}$$

c)

$$\begin{array}{rrrcr} x & +2y & +4z & = & 7 \\ -x & +y & +2z & = & 5 \\ 2x & +y & +2z & = & 2 \end{array}$$

d)

$$\begin{array}{rrrcr} x & +2y & -z & = & -4 \\ 2x & +5y & -z & = & 3 \\ 4x & +9y & -3z & = & -5 \end{array}$$

e) A certain diet should include 1530 mg of calcium, 575 mg of vitamin A and 321 mg of vitamin C. This requirement needs to be fulfilled by drinking milk, orange juice and tomato juice. Milk has 300 mg per cup of calcium, 100 mg per cup of vitamin A and 3 mg per cup of vitamin C. Orange juice has 300 mg per cup of calcium and 90 mg per cup of vitamin C. Tomato juice has 20 mg per cup of calcium, 250 mg per cup of vitamin A and 30 mg per cup of vitamin C. How many cups of each liquid should be taken?

13. **Domains**. Find the domain of:

a) $y = \frac{\sqrt{x-2}}{x^2-9}$

b) $y = \frac{\sqrt{x^2-9}}{x-5}$

c) $y = \sqrt{\frac{(x-3)^3(x+5)}{(x-2)^4}}$

d) $y = \sqrt{x^5 - 2x^4 - 8x^3}$

e) $y = \frac{\log_3(x^2-4)}{x-3}$

f) $y = \frac{x-5}{\ln(x-2)}$

g) $y = \log(\frac{(x-3)^3(x+5)}{(x-2)^4})$

h) $y = \ln(x^5 - 2x^4 - 8x^3)$

14. **Inverse Function**. For given function f find the inverse function f^{-1}.

a) $f(x) = \sqrt[3]{x+1} + 5$

b) $f(x) = \frac{3x-4}{2x+5}$

c) $f(x) = \frac{2x}{x+1}$

d) $f(x) = 2^{3x-1}$

e) $f(x) = \log_3(2x) + 4$

f) $f(x) = \log_5(x-3)$

g) $f(x) = 5\sin\frac{x}{2}$

h) $f(x) = 3\tan(2x + \pi)$.

15. **Composite. Intercepts.** Find the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Find x and y intercepts of $g(x)$.

a) $f(x) = \sqrt{x+2}$ and $g(x) = \ln(x-1)$.

b) $f(x) = x^3$ and $g(x) = (\ln x) - 2$.

c) $f(x) = x^2$ and $g(x) = \log_2(x+3)$.

16. **Rational Functions.** Find the poles and the horizontal and vertical asymptotes of the following rational functions.

a) $f(x) = \frac{x-1}{x+1}$

b) $f(x) = \frac{x-2}{(x-1)(x+3)}$

c) $f(x) = \frac{(x-1)(x+3)}{x-2}$

d) $f(x) = \frac{(2x-1)(x-4)}{(3x+2)(x-1)}$

e) $f(x) = \frac{x}{x^2-4x}$

17. **Linear Modeling.**

a) How many cubic centimeters of water should be added to 480 cubic centimeters of a 2% saline solution so as to decrease the salt concentration to 1.5%

b) How many liters of 75% acid solution must be mixed with 15 liters of 45% acid solution to produce 70% acid solution?

c) How many liters of 60% and 80% acid solution must be mixed together to get 12 liters of 75% acid solution?

d) The life span of an insect can be modified by the temperature of the environment. Assume that the relationship between temperature of the environment (in degrees Celsius) and life span of the fruit flies (in days) is linear. If a population of fruit flies has the life span of 80 days at the temperature of 10 degrees and the life span of 50 days at the temperature of 20 degrees, write the linear relationship between the temperature and the life span. What is the life span at the temperature of 25 degrees? At what temperature is the life span 92 days?

e) Lehigh 2% reduced fat milk can be used 10 days after opening if it is stored at 40 degrees Fahrenheit and 26 days after opening if it is stored at 32 degrees Fahrenheit. Assume that the number of days that fresh milk stays unspoiled depends linearly on the temperature at which milk is kept. Write down the above linear relationship. How long will the milk stay unspoiled at 34 degrees? If the milk is supposed to last 30 days, at what temperature should it be stored?

f) A simple thermometer costs \$10 and the cost of preparing it for the next patient is \$2. A more sophisticated thermometer costs \$120 but it costs only \$.75 to prepare it for the next patient. How many patients would be required for the cost of two thermometers to be equal? Which thermometer is more efficient if the number of patients is large?

g) Acme Products sells a machine for doing a certain type of blood test for \$40,000 which costs \$20 for each use. Amalgamated Medical Supplies sells a similar machine for \$32,000 but it costs \$30 for each use. How many times must the machines be used for the cost to be equal? Which machine is more efficient if it is used many times?

18. **Exponential Growth and Decay.**

a) A bacteria culture grows by the exponential model $y = 200e^{kt}$. How many bacteria are there initially? If the number of bacteria triples in 2 hours, find the number of bacteria after 5 hours.

b) The half-life of bismuth 210 is 5 days. How many days it will take the 1.5 grams of bismuth 210 to decay to 0.3 grams?

c) Suppose that a population grows by 5% per year. Find the time it would take for the population to triple.

d) Happyville and Smileytown both have a population of 10,000 people presently. Happyville is increasing by 1500 people a year and Smileytown is increasing by 15% a year. a) Which town is growing faster? b) Find formulas for the populations of these towns as function of time t in years. c) Use part b) to predict the size of both towns 5 years from now. d) Find a year in which population of Happyville will be over 25000. Do the same for Smileytown.

19. **Other Function Applications.**

a) The number of milligrams of cholesterol consumed each day per person in the United States can be modeled by $0.11x^2 - 4.04x + 445.02$, $1 \leq x \leq 23$ where x represents the number of years since 1974 and $f(x)$ represents the amount of cholesterol consumed each day per person. How many milligrams per person is consumed in 1982? In what year is the amount of cholesterol consumed each day per person minimal?

b) An object is thrown upward. Its height after t seconds is given by $h(t) = 32t - 16t^2$ where the height is given in feet. What is the maximum height of the object? At what time will object hit the ground?

c) The weight (in grams) of a human brain during the last trimester of gestation and the first two years after birth can be approximated by function $w(x) = \frac{x^3}{100} - \frac{1500}{x}$ where x is circumference of the head in cm. What is the approximate weight of brains with a circumference of 30 cm? If an infant brain weights 700 g, what is the circumference of the head?

d) The pH of a liquid is defined as $pH = -\log[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per liter. Find the hydrogen ion concentration for a pH of 5.2. If $9.98 \cdot 10^{-8}$ is the the hydrogen ion concentration for blood, find the pH.

e) Suppose that the spread of a flu at a school with 2000 students total is given by $f(t) = \frac{2000}{1+1999e^{-.8t}}$ where t represent the number of days and f represents the number of students infected. How many students are initially infected? How many students are infected after a week? After how many days will 45% of the total number of students be infected?

20. **Regressions.**

a) A physician decides to measure the healing process of the area of a wound over a 5 day period and collects the following data

time (days)	0	1	2	3	4
area (mm^2)	200	174.3	148.1	122.5	92.7

Find a linear model. Interpret the slope in the context of the problem. When will the wound be completely healed?

b) A company decides to develop a cost equation based on the quantity of the product produced in a day. They collected the following data:

quantity produced	20	35	50	65	80	95	110
cost	642.35	766.48	858.82	928.83	1005.32	1078.82	1140.79

Find a linear and quadratic model for this data. Which model is better? According to the model, how much will producing 195 units cost the company? How many units could be produced for $800?

c) After the winter break, 3 students came to school sick with the flu. The following table shows the number of students infected with the flu depending on the number of days after the winter break.

time (days)	0	5	10	15	20	25	30
number of infected students	3	6	14	23	23	21	9

Find the quadratic and quartic model that fit this data. Which model appears to better fit the data? Using the better model, find the day at which the number of infected students will reach the maximum. When will the number of infected students drop to zero?

d) A new drug was put on the market in 1990. The table below shows the number of prescriptions written for this drug over a 10 year period.

year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
numb. of prescr.	142	149	154	155	159	161	163	164	164	166

Find a logarithmic model for this data. Using the model, how many prescriptions will be written in 2006? In what year will there be 178 prescriptions?

e) The table below shows the yield (in mg) of a chemical reaction in the first 6 minutes.

time (minutes)	1	2	3	4	5	6
yield (mg)	1.2	6.9	9.3	12.7	14.1	15.7

Use the scatterplot to find the best model to fit this data. Using that model, determine in how many minutes will the yield be 20 mg.