MATH 101
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Power, polynomial, and rational functions. Domain of irrational functions.

1. **Power functions.**

   a) If a regression line for $\ln y$ as a function of $\ln x$ has slope 3 and y-intercept 1.5, which power function is $y(x)$ equal to?

   b) If a regression line for $\ln y$ as a function of $\ln x$ has slope -2 and y-intercept -0.5, which power function is $y(x)$ equal to?

   c) Express the power function $y = 5\sqrt{x}$ using $\ln y$ as dependent and $\ln x$ as independent variable.

   d) Find the power model for the following data

x	1	2	3	4	5
y	2.2	51.7	338.9	1236.4	3177.8

2. **Polynomials.**

   a) Test the following data to see whether the data are quadratic.

x	1	2	3	4	5
y	0	3	10	21	36

   b) If the graph of a polynomial has one maximum and two minima, what is the smallest possible degree of that polynomial? If a polynomial has large positive values when x is large, what can you say about the leading coefficient?

   c) If the graph of a polynomial has two maxima and two minima, what is the smallest possible degree of that polynomial? If a polynomial has large negative values when x is large, what can you say about the leading coefficient?

3. **Rational Functions.** Find the poles and the horizontal and vertical asymptotes of the following rational functions.

   (a) $f(x) = \frac{x-1}{x+1}$

   (b) $f(x) = \frac{x-2}{(x-1)(x+3)}$

   (c) $f(x) = \frac{(x-1)(x+3)}{x-2}$

   (d) $f(x) = \frac{(2x-1)(x-4)}{(3x+2)(x-1)}$

   (e) $f(x) = \frac{x}{x^2-4x}$

   (f) $f(x) = \frac{x^2-9}{x-3}$

4. **Domains**. Find the domain of:

   (a) $y = \frac{x-2}{\sqrt{x^2-9}}$

(b) $y = \frac{\sqrt{x^2-9}}{x-5}$

(c) $y = \sqrt{\frac{(x-3)^3(x+5)}{(x-2)^4}}$

(d) $y = \sqrt{x^5 - 2x^4 - 8x^3}$

Solutions.

1. Power Functions.
a) $\ln y = 1.5 + 3\ln x$ means that $y = e^{1.5}x^3 = 4.48x^3$ b) $\ln y = -.5 - 2\ln x$ means that $y = e^{-.5}x^{-2} = \frac{.607}{x^2}$ c) $y = 5\sqrt{x}$ implies that $\ln y = \ln 5 + \frac{1}{2}\ln x = 1.61 + \frac{1}{2}\ln x$ d) $y = 2.23x^{4.54}$

2. Polynomials.
a) Second-order difference is constant (4) so the data are quadratic. Quadratic function that fits the data is $y = 2x^2 - 3x + 1$. b) Degree 4, leading coefficient positive. c) Degree 5, leading coefficient negative.

3. Rational Functions.

   a) Horizontal $y = 1$, vertical (and pole) $x = -1$.
   b) Horizontal $y = 0$, vertical (and poles) $x = 1$, and $x = -3$.
   c) No horizontal, vertical and pole $x = 2$.
   d) Horizontal $y = 2/3$, vertical (and poles) $x = 1$ and $x = -2/3$.
   e) Horizontal $y = 0$, $x = 0$ is a pole but not a vertical asymptote. $x = 4$ is both a pole and a vertical asymptote.
   f) No horizontal and vertical asymptotes. $x = 1$ is a pole.

4. Domains. a) $x > 3$ or $x < -3$ b) $x \geq 3,\ x \leq -3$ or $x \neq 5$ c) $x \leq -5$ or $x \geq 3$ d) $-2 \leq x \leq 0$ or $x \geq 4$