MATH 101
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Piecewise-defined Functions

1. Graph $y = \begin{cases} 1 & x > 1 \\ x+1 & x \leq 1 \end{cases}$ Evaluate $f(-2)$, $f(1)$, $f(2)$.

2. Graph $y = \begin{cases} 2-x & x \geq 1 \\ x & -1 \leq x < 1 \\ -x-2 & x < -1 \end{cases}$ Evaluate $f(-2)$, $f(1)$, $f(3)$.

3. Graph $y = \begin{cases} x+2 & x < -2 \\ 0 & -2 \leq x < 2 \\ (x-2)^2 & x \geq 2 \end{cases}$ Evaluate $f(-3)$, $f(1)$, $f(3)$.

4. A tomato plant grows linearly by 2 inches every month starting from the height of 5 inches. a) Write down an equation representing the relationship between the height of the plant and the number of month that passed since the plant was 5 inches tall. b) After 3 months (how tall is the plant then?), the neighboring rose bush starts obstructing the sunlight the tomato plant is getting. This results in the change of the rate of growth of the tomato plant - the growth drops from 2 to 1.5 inches every month. Write down the piecewise linear function that describes the height of the tomato plant as a function of the number of months passed. Sketch this function. c) How many months need to pass before the plant is 20 inches tall?

5. The concentration of a medication in patients body increases linearly during the first two hours. Initially, it is 3 μg/ cm^3, and after two hours it is 3.25 μg/ cm^3. After two hours, the concentration starts decreasing so that 5 hour after, it is 2.80 μg/ cm^3. Write a piecewise linear function that describes the concentration of medication as a function of number of hours passed. Sketch this function. When will the concentration drop below 2 μg/ cm^3?

6. Initially there is 50 mg of bacteria culture. The number of bacteria is doubling every hour for the first five hours. What is the bacteria size after 5 hours? After 5 hours, the growth rate slows down and the culture increases by 60 mg every hour. Write down a piecewise function that describes the bacteria size as function of time in hours. Sketch the function. Use the formula to find the bacteria size 4 and 6 hours after the start of experiment.

7. The size of a population of rabbits in a certain habitat is described by a table below.

year	2000	2001	2002	2003	2004	2005	2006
number of rabbits	30	45	68	60	96	154	247

We can see that the number is increasing from 2000 to 2003. There was a decrease in number of rabbits in 2004 due to a flood in the habitat but after 2004, the number of

rabbits is increasing again. Assuming that the number of rabbits is increasing exponentially both before the flood and after the flood, find the two exponential regressions that will best fit the data before and after the flood. Using the two formulas, write down a piecewise function that will describe the number of rabbits from 2000 to 2006. Estimate the number of rabbits in 2010.

Solutions.

1. $f(-2) = -1$, $f(1) = 2$, $f(2) = 1$.

2. $f(-2) = 0$, $f(1) = 1$, $f(3) = -1$.

3. $f(-3) = -1$, $f(1) = 0$, $f(3) = 1$.

4. a) $y = 5 + 2x$. b) $y = \begin{cases} 5 + 2x & x \leq 3 \\ 6.5 + 1.5x & x > 3 \end{cases}$ c) 9 months.

5. $y = \begin{cases} 3 + .125x & x \leq 2 \\ 3.55 - .15x & x > 2 \end{cases}$ The concentration will drop below 2 μg/ cm^3 10.333 hours (or 10 hours and 20 minutes) after the medication is given.

6. Before 5 hours, the exponential function has initial size 50 and the growth rate 2, so it is $y = 50(2)^t$ for $t \leq 5$. When $t = 5$, $y = 1600$. After 5 hours, the function is changing to a line with slope 60, passing (5, 1600). So, it is $y = 60t + 1300$ for $t > 5$. Thus, the piecewise function is $y = \begin{cases} 50(2)^t & t \leq 5 \\ 60t + 1300 & t > 5 \end{cases}$ $y(4) = 800$ mg and $y(6) = 1660$ mg.

7. The exponential regression for the first three given points gives us the formula $y = 29.96(1.5055)^x$. The exponential regression for the next four points gives us $y = 14.56(1.6028)^x$. Thus, the piecewise function is $y = \begin{cases} 29.96(1.5055)^x & 0 \leq x \leq 2 \\ 14.56(1.6028)^x & x \geq 3 \end{cases}$ $y(10) = 1629.47 \approx 1629$ rabbits.