MATH 101
Lia Vas

Logarithmic and Exponential Functions

1. a) Find an exponential function with growth factor 2 passing point (3, 6).
   b) If a certain phenomenon grows by 7% each year and has initial value 10, find an exponential function describing it.
   c) If a certain phenomenon continuously grows by 7% and has initial value 10, find an exponential function describing it.
   d) Test if following data is linear, exponential or neither.

x	2	4	6	8
y_1	3	9	27	81

x	2	4	6	8
y_2	3	9	81	6561

x	2	4	6	8
y_3	3	12	21	30

2. Solve for x.
   a) $\log_5(x^2+9) = 2$
   b) $e^{4x-8} = 1$
   c) $\ln(x+2) + \ln e^3 = 7$
   d) $e^{e^x} = 4$
   e) $2\ln x - 2\ln 4 = \ln(x-3)$
   f) $3^{x+2} = 7$

3. Find the inverse $f^{-1}(x)$ for the given function $f(x)$.
   a) $f(x) = 12^x$
   b) $f(x) = \log_3 x$
   c) $f(x) = \ln x - 2$
   d) $f(x) = e^{2x} + 5$
   e) $f(x) = \log_5(x-3)$
   f) $f(x) = 2^{3x-1}$
   g) $f(x) = \log_3(2x) + 4$

4. Find the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Find x and y intercepts of $g(x)$.
   a) $f(x) = \sin x$ and $g(x) = 2^x - 3$.
   b) $f(x) = \sqrt{x+2}$ and $g(x) = \ln(x-1)$.
   c) $f(x) = x^3$ and $g(x) = \ln x - 2$.
   d) $f(x) = x^2$ and $g(x) = \log_2(x+3)$.

Solutions.

1. a) $y = y_0 2^x$. Since $6 = y_0 2^3$, $y_0 = 6/8 = 3/4$. Hence, $y = \frac{3}{4}2^x$.
   b) $y = 10(1 + .07)^t = 10(1.07)^t$
   c) $y = 10e^{.07t} = 10(1.0725)^t$.
   d) y_1 is exponential with growth factor 3. y_2 is neither. y_3 is linear with growth rate 9.

2. a) $x = \pm 4$
   b) $x = 2$
   c) $x = e^4 - 2 = 52.6$
   d) $x = \ln(\ln 4)) = .33$
   e) $x = 12$ and $x = 4$
   f) $x = -.229$

3. a) $f^{-1}(x) = \log_{12} x$
   b) $f^{-1}(x) = 3^x$
   c) $f^{-1}(x) = e^{x+2}$
   d) $f^{-1}(x) = 1/2 \ln(x - 5)$
   e) $f^{-1}(x) = 5^x + 3$
   f) $f^{-1}(x) = 1/3(\log_2 x + 1)$
   g) $f^{-1}(x) = \frac{3^{x-4}}{2}$

4. a) $(f \circ g)(x) = \sin(2^x - 3)$ $(g \circ f)(x) = 2^{\sin x} - 3$. y-intercept of $g(x)$= (0,-2), x-intercept of $g(x)$= $(\log_2(3), 0) = (1.58, 0)$.
   b) $(f \circ g)(x) = \sqrt{\ln(x-1) + 2}$, $(g \circ f)(x) = \ln(\sqrt{x+2} - 1)$. No y-intercept of $g(x)$, x-intercept=(2,0).
   c) $(f \circ g)(x) = (\ln(x) - 2)^3$, $(g \circ f)(x) = \ln(x^3) - 2$. No y-intercept of $g(x)$, x-intercept $= (e^2, 0) = (7.39, 0)$.
   d) $(g \circ f)(x) = (\log_2(x+3))^2$, $(f \circ g)(x) = \log_2(x^2+3)$. y-intercept of $g(x) = (0, \log_2(3)) = (0, 1.58)$. x-intercept = (-2,0).