MATH 101
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Hardy-Weinberg Theorem

Let us calculate the percent of the right-handed people in this class that are carrying left-handed genes (i.e. have a predecessor who was left-handed and might have left-handed children). The properties of being left/right handed are being carried by the part of the chromosomes called genes. An allele is the particular variation of a gene that determines the genetic makeup of an organism (in this case whether you are right- handed or left-handed). For right/left handedness, there are two alleles

A dominant allele – carries right-handedness,

a recessive allele – carries left-handedness.

In each cell, chromosomes come in pairs, so the possible combinations are: AA, Aa, and aa.

Since A is dominant, people with combinations AA or Aa in their cells are right-handed. People with aa are left-handed.

Some terminology:

AA or Aa – dominant phenotype.

Aa – carriers of recessive trait. AA – non-carriers.

aa – recessive phenotype.

Homozygous – having pair of identical alleles (in this case AA or aa).

Heterozygous – having two different alleles (Aa).

Frequency – number of cases of particular event / total number of cases.

p = frequency of A

q = frequency of a

Note:

$$p + q = 1 = 100\%$$

p^2 = frequency of AA

q^2 = frequency of aa

$2pq$ = frequency of Aa

Since $p + q = 1$,

$$1 = (p + q)^2 = p^2 + 2pq + q^2$$

Hardy – Weinberg Theorem

Examples.

1. Suppose there are 30 students in a class and 3 are left-handed. How many right-handed students carry a recessive allele?

   Solution: The frequency of aa is 3/30 students. So, $q^2 = 3/30 = 0.1$. Hence, $q = .316$. From that we get that $p = 1-q = .684$. To find how many students carry a recessive allele, we need to find the frequency of recessive allele $2pq$. $2pq = 2 \cdot .684 \cdot .316 = .432 = 43.2\%$. So, the number of right-handed students who carry a recessive allele is 43.2% of 30 = .432 x 30 = 13 students. Note: The frequency of right-handed students (AA or Aa) is $27/30 = 0.9 = p^2 + 2pq = 1 - q^2$.

2. Experiments have been performed in which you find that 41% of the population displays the recessive trait. Determine the percentage of the population that are carriers of the recessive trait? That are homozygous?

   Solution: $q^2 = 41\% = .41$. So $q = .6403$. Hence $p = 1 - q = .3597$. The problem is asking for percentage of carriers of recessive trait = heterozygous. So it is $2pq = .4606 = 46.06\%$. Non-carriers: $p^2 = .1294 = 12.94\%$. Homozygous: $p^2+q^2 = .1294+.41 = .5394 = 53.94\%$.

3. If the frequency of the recessive allele is .32, what percentage of population display the dominant phenotype?

   Solution: Frequency of the recessive allele = $q = .32$. So $p = 1 - q = .68$. The problem is asking for the percentage of dominant phenotype = $p^2 + 2pq = .8976 = 89.76\%$.

Practice Problems.
Let us assume that the following populations are in Hardy-Weinberg equilibrium.

1. If 16% of students in a group are left-handed, what is the percentage of the students that are heterozygous? Homozygous?

2. If 84% of the population displays a dominant trait, what is the frequency of dominant allele?

3. If frequency of the recessive allele is 20%, what is the percentage of those in the population displaying the dominant phenotype? What is the percentage of those displaying dominant phenotype that are carriers of recessive allele? What is the percentage of those in the population displaying the recessive phenotype?

4. If 91% of students in a class are tongue rollers, what is the percentage of the students that are not carriers of recessive trait?

5. If 4% of a population displays a recessive trait, what is the percentage of those that display the dominant trait but carry the recessive allele?

Solutions:

1. Heterozygous 48%. Homozygous 52%. 2. 0.6

3. 96% display dominant phenotype. 32% display dominant phenotype and carry recessive allele. 4% display recessive phenotype. 4. 49% 5. 32%