Hardy-Weinberg Theorem and Blood Types

Recall that the Hardy - Weinberg Theorem says that $p+q = 1$ and

$$1 = (p+q)^2 = p^2 + 2pq + q^2$$

where *A* = dominant allele, *p* = frequency of *A*, *a* recessive allele, *q* = frequency of *a.* We use this formula when there are just **two** choices of alleles in a gene: dominant *A* or recessive *a*.

In some cases, there might be **three** choices of alleles. For example, 4 blood types *A, B, AB* and *O* are determined by **three** possible choices of alleles denoted by: ***A, B, O***. **Alleles *A* and *B* are dominant over *O*.**

In each cell, alleles come in pairs, so the possible combinations are: *AA, AB=BA, AO=OA, BB, BO=OB* and *OO*.

A person will have **blood type *A*** if the combinations *AA* or *AO=OA* occur. The combination *AO* (same as *OA*) will result in blood type *A* because the allele *A* is dominant over allele *O*.

A person will have **blood type *B*** if the combinations *BB* or *BO=OB* occur. The combination *BO* (same as *OB*) will result in blood type *B* because the allele *B* is dominant over allele *O*.

A person will have **blood type *AB*** if the combination *AB=BA* occurs. The alleles A and B are "equally strong" so they yield a new blood type *AB*.

A person will have **blood type *O*** if the combination *OO* occurs.

	A	*B*	*O*
A	*AA=A*	*AB*	*OA=A*
B	*AB*	*BB=B*	*OB=B*
O	*OA=A*	*OB=B*	*OO=O*

As before, a **homozygote** is an organism with pair of identical alleles. In this case, this is *AA* or *BB* or *OO*. A **heterozygote** is an organism with two different alleles. In this case, this is *AB, AO,* or *BO*.

Recall that, the frequency = number of cases of particular event / total number of cases.

Let us denote:
p = frequency of A
q = frequency of B
r= frequency of O.

Thus

$$1 = p + q + r$$

and the Hardy-Weinberg Theorem in this case will be

$$1 = (p + q + r)^2 = (p + q + r)\ (p + q + r) = p^2 + pq + pr + qp + q^2 + qr + rp + rq + r^2 =$$

$$p^2 + q^2 + r^2 + 2pq + 2pr + 2qr = 1$$

Hence:

Blood Type	O	A	B	AB
Frequency	r	$p+2pr$	$q+2qr$	$2pq$

Examples:

1. A study of the blood types in France produced information in the table below. What fraction of type A individuals are AO heterozygotes? Determine the frequencies of the alleles A, B, and O.

Blood Type	O	A	B	AB
Frequency	.441	.435	.090	.034

(taken from http://www19.homepage.villanova.edu/alice.deanin/courses/Mat7310/Hardy%20Weinberg.htm)

Solution: The first part of the problem is asking us to find $2pr$. We have that r^2=.441. Thus, r = .664. The frequency of type A is .435 and so $p+2pr$= .435. This gives us a quadratic equation $p+1.328\ p$ -.435=0. Solve it using your quadratic equation program. Get .272 and -1.6. We discard the negative answer and so p=.272. Finally, we get that $2pr$=.361.

The second part of the problem is asking us to find p, q and r. From the first part, we have that r = .664 and p=.272. Then we can get q from q=1-p-r=1-.664-.272= .064.

2. In Zurich, Switzerland, the allele frequencies of A, B, and O are 0.27, 0.06, and 0.67, respectively. Find the percentage of people with blood types A, B, AB, and O?
(taken from http://www.mansfield.ohiostate.edu/~sabedon/hardy_weinberg_practice_problems.htm)

Solution: p=0.27, q=0.06 and r=0.67.

Type $A = p+2pr = 0.4347$.
Type $B = q+2qr = 0.084$.
Type $AB = 2pq = .0324$.
Type $O = r^2 = .4489$.

Practice Problems

1. If 45% of people within a population have blood type *O,* and 40% of population have blood type *A,* find the percentage of people with blood types *B* and *AB*. Note: this data matches the **world distribution of blood types**. Source:
http://www.all-science-fair-projects.com/science_fair_projects_encyclopedia/Blood_type

2. Find the percentage of homozygous people with the data from previous problem.

3. If 49% of people within a population have blood type *O,* and 7.25% of population have blood type *B,* find the percentage of people with blood types *A* and *AB*.

4. (Extra Credit) If 45% of people within a population have blood type *O,* and 3% of population have blood type *AB,* find the percentage of people with blood types *A* and *B*.

Solutions:

1. First find *r*, then *p* and then *q*. Get $r=.671$, $p=.251$, $q=.078$. Type *B* = 11%. Type *AB* = 3.92% approximately 4%.

Blood Type	*O*	*A*	*B*	*AB*
world frequency	45%	40%	11%	4%

2. 52%

3. $r=.7$, $q=.05$, $p=.25$. Type *A* = 41.25%. Type *AB* = 2.5%.

4. $r=.671$. To find *p* and *q*, solve the following system of equations: $p+q=.329$ and $2pq=.03$. Get solutions .055 and .274. Then $p=.274$, $q=.055$ (note that type *A* is much more frequent than type *B*). Then, type *A* = 44.3% approximately 44%. Type *B* = 7.7% approximately 8%.