MATH 101
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Functions by Tables. Functions by Graphs.

Functions by Tables.

The average rate of change tells you how much did the function values changed on average. The average rate of change of function $y = f(x)$ on interval $[x_1, x_2]$ is

$$\frac{\text{difference of } y\text{-values}}{\text{difference of } x\text{-values}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Units: units of y per units of x. Applications: average velocity.

Filling gaps by averaging. Suppose that (x_1, y_1) and (x_2, y_2) are two points of a function given by a table and that the function value should be approximated at a point x_m close to the middle of the interval (x_1, x_2). In the absence of more information, a reasonable guess is the average of y-values at x_1 and x_2.

$$y(x_m) \approx \frac{y_1 + y_2}{2}$$

Using average value to fill the gaps. If (x_1, y_1) and (x_2, y_2) are two points of a function and if the function value should be evaluated at a point x_m, one can use average value.

1. Find the average value $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Approximate the y-value by adding the product of the average rate and the change in x-values to the initial y-value.

   $$y_m \approx y_1 + m(x_m - x_1).$$

   Note that $x_m - x_1$ might be negative.

Practice Problems.

1. Let

x	0	5	10	15	20
$f(x)$	20	26	31	33	39

   a) What is the average rate of change from $x = 5$ to $x = 10$? b) Use your answer to estimate value at $x = 7$. c) What is the average rate of change from $x = 15$ to $x = 20$? d) Use your answer to estimate value at $x = 18$. e) Use your answer to estimate value at $x = 22$.

2. In an experiment, a bacteria culture is monitored and its size (in mg) is being recorded every 10 hours. Let

time (hours)	0	10	20	30	40	50	60
size (mg)	17.6	23.8	44.6	51.3	53.2	53.7	53.9

a) What is the average rate of change from 20 to 30 hours? Explain what that answer means. b) Use your answer to estimate the size 23 hours after the start of experiment. c) What is the average rate of change from 30 to 40? d) Use your answer to estimate the size 37 hours after the start of experiment. e) What is the expected size of this bacteria culture on the long run? (i.e. what is the limiting value of the size?)

3. Carbon 14 is a radioactive substance that decays over time. In the following table, time t is measured in thousands of years and C is the amount in grams of carbon 14 remaining.

time (thousands of years)	0	5	10	15	20
amount remaining (grams)	5	2.73	1.49	0.81	0.44

a) What quantity of carbon 14 will be present after 10,000 years? b) What is the average rate of change of the amount of carbon 14 between 10,000 and 15,000 years? Explain what the sign of your answer means. Include the units. c) Use the answer from b) to estimate the amount of carbon 14 after 12,000 years. d) How many grams of carbon 14 do you expect to find after a very large number of thousands of years?

Functions by Graphs.

1. The following set of questions relates to the function given by a graph in problem S1, section 1.3, page 62 of your textbook. a) Determine the smallest value of x for which $f(x) = 1.5$ b) Where does graph reach a maximum and what is the maximal value? c) Where is the graph increasing? Decreasing? d) What is the concavity for $1.8 \leq x \leq 3$? e) Estimate a point at which the rate of increase is maximal. f) Estimate a point at which the rate of decrease is maximal. g) What are the inflection points?

2. The following set of questions relates to the function given by a graph in problem 5, section 1.3, page 63 of your textbook. a) Estimate the flow at the end of June. b) Estimate the time(s) when the flow is 1200 cubic feet of water per second. c) Estimate the time when the flow is decreasing the fastest. d) Estimate the average rate of change from beginning of November to end of December. Explain what your answer means for the flow of the Arkansas river.

3. The following set of questions relates to the function given by a graph in problem 7, section 1.3, page 64 of your textbook. a) Estimate the net stumpage value of a Douglas fir stand that is 60 years old. b) Estimate the age of a Douglas fir stand whose net stumpage value is $40,000 per acre. c) At what age does the commercial value of the stand equal equal the cost of felling, hauling, etc.? d) At what age is the net stumpage value increasing the fastest?

Solutions.
Functions by tables.

1. a) 1 b) 28 c) 6/5 d) 36.6 e) 41.4

2. a) Average rate = .67. This means that the size of bacteria is increasing by .67 mg every hour between 20 and 30 hours after the experiment started. b) 48.27 mg c) Average rate = .19. This means that the size of bacteria is increasing by .19 mg every hour between 30 and 40 hours after the experiment started. d) 52.63 mg e) 54 mg.

3. a) From table $y = 1.49$. So, 1.49 grams. b) $\frac{.81-1.49}{15-10} = -.136$ grams per thousand years. Thus, the amount is decreasing by .136 grams every 1000 years. c) $1.49 - .136(12 - 10) = 1.218$ grams d) 0 grams.

Functions by graphs.

1. a) x close to 1. Max is reached at $x = 2.4$. Max value is about 4. c) The graph is increasing for $0 < x < 2.4$ and decreasing for $2.4 < x < 4.8$. d) Concave down. e) $x = 1.5, y = 2.7$ f) $x = 3.3, y = 2.7$ g) (1.5, 2.7) and (3.3, 2.7).

2. a) about 2200 cubic feet per second. b) When is little less than $x = 5$ and $x = 8$ approximately. This corresponds to late May and late August. c) When $x = 7$ approximately, so late July to early August. d) Values at $x = 10$ and $x = 12$ are approximately equal. The average rate of change for two equal y-values is 0. That means that the flow is changing at a constant average rate during those months (careful: zero rate does not mean that the flow is zero – it means that the flow is constant)

3. a) 14,000 dollars per acre. b) At about 110 years. c) At about 30 years. d) At about 65 years.